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An experiment on specific heat of solids

H W JONES  Physics Department, University of Calgary, Canada
B MORGAN  Fitzroy High School, Melbourne, Australia

The discoveries in early low temperature physics research and the subsequent theoretical treatments of the variations of specific heat with temperature made an exciting chapter in the history of physics. We hope some of this interest can be brought into the laboratory by an experiment in this area. Liquid nitrogen is readily available in many places and makes it possible to work at reasonably low temperatures. This, combined with the fact that there are two common solids, aluminium and carbon, which have low specific heats at these temperatures, prompted us to design an experiment for undergraduates.

Description

Nernst (1910, 1911) used a vacuum insulated calorimeter to determine specific heats at low temperatures and the major features of his design have been incorporated in the present apparatus. Figure 1 shows the general arrangement. An aluminium bobbin is wound (non-inductively) with a (single layer) coil of enamel insulated 0.006 in diameter platinum wire (giving a resistance of about 20 \( \Omega \) at room temperature). The bobbin is suspended from its leads via a PTFE plastic top in a glass enclosure and rests on three dimples in the glass tube. The calorimeter is used in conjunction with the circuit shown in figure 2. A bridge circuit is used to measure temperature via the resistance of the coil. The same coil is used for heating the specimen. Figure 3 shows an apparatus that was used in the laboratory.

The success of the experiment depends both on satisfactory design and choice of materials. In principle there are only two materials for the calorimeter specimen which are of interest: aluminium and carbon. Both satisfy two necessary conditions in that they have a sufficiently large change of specific heat over the temperature range of interest and a satisfactory thermal diffusivity, as is shown in table 1.

Graphite presents difficulties which are well described in the words of Bergenlid et al (1954):

> 'Firstly, owing to its small specific heat, the heat capacity of a specimen is rather small compared with that of a suitable calorimeter. The second difficulty is that graphite is a powerful absorbent for gases whose relative heat capacity may be considerable at low temperatures'. Our experience confirmed the difficulties with this material and led us to reject it.

Further information on the use of graphite in this regard will be found in DeSorbo and Tyler (1953).

Secondly, concerning cooling losses, a compromise between two conflicting requirements is needed. The calorimeter must be able to cool down from room to liquid nitrogen temperatures in a relatively short length of time. However, after it has reached liquid nitrogen temperature and is reheated by the electric current, the heat losses must not be so great that a satisfactory cooling curve cannot be obtained during the experiment (figure 4). If the aluminium specimen is in an evacuated envelope (to better than 10^{-3} Torr) then cooling from room temperature takes about 24 hours, which is too long. Several gas fillings were tried and in the end, the calorimeter with air at atmospheric pressure was found to be as effective as any other. With this arrangement it is possible to obtain equilibrium in about one hour, after completely immersing the calorimeter in liquid nitrogen. Figure 4 shows that there is still ample time to make the measurements required at liquid nitrogen temperatures.

<table>
<thead>
<tr>
<th>Material</th>
<th>Specific heats (cal deg^{-1}g^{-1})</th>
<th>Thermal diffusivity</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Room temperature</td>
<td>Liquid nitrogen temperature</td>
</tr>
<tr>
<td>Aluminium</td>
<td>0.21</td>
<td>0.08</td>
</tr>
<tr>
<td>Graphite</td>
<td>0.17</td>
<td>0.03-0.025</td>
</tr>
</tbody>
</table>

See following references: Giaugue and Meads (1941), Berman (1952), Smith (1954), Rosenberg (1955), DeSorbo and Tyler (1953), Carslaw and Jaegen (1958), Bergenlid et al (1954)
The temperature is measured by a Wheatstone bridge (figure 2). The bridge is balanced when the calorimeter has reached its equilibrium temperature in liquid nitrogen. The fact that an equilibrium has been achieved is easily checked by taking several readings of the bridge, a few minutes apart. The current flowing through the coil when a temperature reading is being made is about 15 mA, consequently the heating rate due to this is only $8.5 \times 10^{-3}$ W or thereabout, which is insignificant.

In one application we decided to use the out of balance current to measure temperature. For this reason, some calorimeters were calibrated with a particular galvanometer and a certificate was attached to the apparatus. In a more extensive experiment the resistance thermometer can be calibrated against fixed points.

It should be mentioned that the size of the aluminium sample was to some degree decided by the choice of the liquid nitrogen flask available; a cheap picnic flask. It can be shown that reasonably accurate results can be obtained with a mass of about 12 g.

Typical of the results, which were observed by the students, are as follows:

**TYPICAL DATA**

As obtained by students: Specimen — aluminium

Heating current ($I$) = 0.54 A (from 2.3 V for 20 s)

Mass of specimen ($m$) = 12.47 g

Temperature rise (figure 4) $\Delta\theta = 5.75 \, ^\circ C$

Calculated specific heat $C_p = 0.083 \, \text{cal} \, g^{-1} \, \text{deg}^{-1}$

that is

$$C_p = \frac{IV}{4.182 \, m \Delta\theta}.$$

**IMPORTANT ERRORS**

$I = \pm 1\% \quad V = \pm 2\% \quad \Delta t = \pm 2\% \quad \Delta\theta = \pm 3\%$

Total error could be as high as 8%.

We note that a 2% error would arise from the presence of the platinum wire.

If the out of balance current is used to measure temperatures, then a correlation of the galvanometer reading with temperature of the specimen is required.

Writing

$$\frac{dl_0}{dr} = V \frac{d}{dr} \left( \frac{R_1 R_4 - R_2 R_3}{R_1 + R_2} \right)$$

where

$$Y = \begin{vmatrix} -R_4 & R_4 + R_3 & R_3 \\ -R_4 & R_1 + R_3 & R_3 + R_4 \\ R_1 & R_1 + R_3 & R_3 + R_4 \end{vmatrix}$$

as $dR_3/d\theta$ is given by differentiation of a relationship of the form

$$\theta = A_1 R_\theta - A_2 R_\theta^2 + \ldots$$

(Robertson and Walsh 1962) then the deflection of the galvanometer is given by:

$$S \left( \frac{dl_0}{dr} \frac{dR_0}{d\theta} \right) = \frac{dl_0}{d\theta}$$

where $R_0$ is the coil resistance at temperature $\theta$, $A_1$
(etc) are constants and $S$ is the galvanometer sensitivity. It follows from equation (3) that only to a first order is the galvanometer deflection directly proportional to the temperature change. In our case, we found that $dl/d\theta$ was about $10.4 \text{ cm}^\circ \text{C}^{-1}$ over a $6 \circ \text{C}$ range, giving an error of $\pm 1\%$ in $\Delta \theta$.

Concerning the temperature difference through the specimen, the diffusion equation is:

$$\frac{\partial^2 \theta}{\partial t^2} = \kappa \nabla^2 \theta.$$  \hspace{1cm} (4)

For simple cylindrical coordinates in which the $z$ dependence is ignored, we have:

$$\frac{1}{\kappa} \frac{\partial \theta}{\partial t} = \left( \frac{\partial^2 \theta}{\partial r^2} + \frac{1}{r} \frac{\partial \theta}{\partial r} \right).$$  \hspace{1cm} (5)

For the case of the calorimeter the heating rate (flux $F_0$) at the outside surface (radius $a$) is a constant. Consequenly:

$$F_0 = -K \frac{\partial \theta}{\partial r} \bigg|_{r=a}.$$  \hspace{1cm} (6)

A solution of equation (4) (Carslaw and Jaeger 1959) under these boundary conditions gives:

$$\theta = \frac{2F_0 \kappa t}{K} + \frac{F_0 a}{K} \times \left[ \frac{r^2}{2a^2} - \frac{1}{4} - \frac{2}{\sum_{n=1}^{\infty}} - \frac{\kappa \alpha_n^2}{a^2} \right] J_0 \left( \frac{\alpha_n a}{a} \right)$$  \hspace{1cm} (7)

where $\alpha_n$, $n=1, 2, \ldots$, are the positive roots of $J_1(\alpha) = 0$.

Figure 5 shows solutions of equation (7) and it can be seen that for practical purposes equilibrium is achieved for $\kappa t/a^2 > 0.2$. For the calorimeter described this means that at times greater than $t=0.02$ s, equilibrium is achieved and the maximum difference between the inside and outside temperatures

$$\Delta \theta = \frac{F_0 a}{2K}$$

would only be $0.01 \circ \text{C}$ (for the data given). It is interesting to know what temperature difference will exist across the specimen during cooling. It can be supposed that heat is lost at a rate directly proportional to the temperature difference between the surface of the specimen and the liquid nitrogen. This being the case then a solution of equation (5) gives:

$$\theta = \frac{2}{a^2} \sum_{n=1}^{\infty} \exp \left( -\kappa \alpha_n^2 t \right) \frac{\alpha_n^2 J_0(\alpha_n a)}{J_0^2(a)}$$

$$\int \phi(r) J_0(\alpha_n a) \, dr$$  \hspace{1cm} (8)

where $\theta = \phi(r)$ at $t=0$ and the temperature variation across the specimen can be found to be about $0.005 \circ \text{C}$, after initial relaxation.

**Conclusion**

The general principles and the design of a fairly simple apparatus for the measurement of the specific heat of aluminium at liquid nitrogen temperatures is discussed. A set of readings taken by students with a typical set of apparatus in use in a teaching laboratory is presented.
Figure 4 Cooling curve. At 2.98 cm deflection and $t=0$, heating period = 20 s, $\Delta r = (2.98 \times 20)/10.37 = 5.75^\circ$C. Galvanometer number 12 $\times 0.05$ range, $dl/dr = 10.37$ cm $^\circ$C$^{-1} \times 1$ range

Acknowledgments

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Figure 5 The temperature in a circular cylinder with constant flux at the surface. The numbers on the curves are the values of $\kappa r/a^2$


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Electronics summer school

The Department of Electrical Engineering Science of Essex University will be holding its annual electronics summer school for teachers during the week of 8–12 July. This will be the third year the summer school has been held and in the light of past experience the curriculum has been updated. Teachers will now be given the choice between courses on linear circuit design and digital circuit design. Although these courses will run simultaneously, they will be repeated in 1975 to enable those interested to study both topics in successive years.

By narrowing the spectrum of material covered in a week, time will be available for a more thorough appreciation of the important aspects of each topic. Both courses will concentrate on the practical application of electronics and laboratory work will occupy approximately 70% of the course time.

The linear circuit design course will include basic semiconductor theory, pre-amplifier and power amplifier design, the use of operational amplifiers and power supply design. The digital circuit design course will cover transistor and diode switching principles, multivibrators, logic circuits, combinational and sequential circuit design and an introduction to digital systems.

Further information can be obtained from the Summer School Coordinator, Mr R J Mack, at the following address: Department of Electrical Engineering Science, University of Essex, Wivenhoe Park, Colchester, Essex CO4 3SQ (telephone: 0206-44144).