The Justification of Deduction

SUSAN HAACK

(1) It is often taken for granted by writers who propose—and, for that matter, by writers who oppose—'justifications' of induction, that deduction either does not need, or can readily be provided with, justification. The purpose of this paper is to argue that, contrary to this common opinion, problems analogous to those which, notoriously, arise in the attempt to justify induction, also arise in the attempt to justify deduction.

Hume presented us with a dilemma: we cannot justify induction deductively, because to do so would be to show that whenever the premisses of an inductive argument are true, the conclusion must be true too—which would be too strong; and we cannot justify induction inductively, either, because such a 'justification' would be circular. I propose another dilemma: we cannot justify deduction inductively, because to do so would be, at best, to show that usually, when the premisses of a deductive argument are true, the conclusion is true too—which would be too weak; and we cannot justify deduction deductively, either, because such a justification would be circular.

The parallel between the old and the new dilemmas can be illustrated thus:

\begin{align*}
\text{Hume's dilemma} & \\
\downarrow \text{induction} & \\
\text{deductive} & \text{inductive} \\
\text{justification} & \text{justification} \\
\text{too strong} & \text{circular} \\
\text{The new dilemma} & \\
\downarrow \text{deduction} & \\
\text{inductive} & \text{deductive} \\
\text{justification} & \text{justification} \\
\text{too weak} & \text{circular} \\
\end{align*}

(2) A necessary preliminary to serious discussion of the problems of justifying induction/deduction is a clear statement of them.

This means, first, giving some kind of characterisation of 'inductive argument' and 'deductive argument'. This is a more difficult task than seems to be generally appreciated. It will hardly do, for example, to characterise deductive arguments as 'non-ampliative' (Salmon [1966]) or

I have profited from comments made when an earlier version of this paper was read to the Research Students' Seminar in Cambridge, May 1972.
'explicative' (Barker [1965]), and inductive arguments as 'ampliative' or 'non-explicative'; for these characterisations are apt to turn out either false, if the key notion of 'containing nothing in the conclusion not already contained in the premises' is taken literally, or trivial, if it is not.

Because of the difficulties of demarcating 'inductive' and 'deductive' inference, it seems more profitable to define an argument:

An argument is a sequence $A_1 \ldots A_n$ of sentences ($n \geq 1$), of which $A_1 \ldots A_{n-1}$ are the premises and $A_n$ is the conclusion —and then to try to distinguish inductive from deductive standards of a 'good argument'.

It is well known that deductive standards of validity may be put in either of two ways: syntactically or semantically. So:

$$D_1 \quad \text{An argument } A_1 \ldots A_{n-1} \vdash A_n \text{ is deductively valid (in } L_D)\text{ just in case the conclusion, } A_n, \text{ is deducible from the premises, } A_1 \ldots A_{n-1}, \text{ and the axioms of } L_D, \text{ if any, in virtue of the rules of inference of } L_D \text{ (the syntactic definition).}$$

$$D_2 \quad \text{An argument } A_1 \ldots A_{n-1} \vdash A_n, \text{ is deductively valid just in case it is impossible that the premisses, } A_1 \ldots A_{n-1}, \text{ should be true, and the conclusion, } A_n, \text{ false (the semantic definition).}$$

Similarly, we can express standards of inductive strength either syntactically or semantically; the syntactic definition would follow $D_1$ but with ' $L_I$ ' for ' $L_D$ '; the semantic definition would follow $D_2$ but with 'it is improbable, given that the premisses are true, that the conclusion is false'.

The question now arises, which of these kinds of characterisation should we adopt in our statement of the problems of justifying deduction/induction? This presents a difficulty. If we adopt semantic accounts of deductive validity/inductive strength, the problem of justification will seem to have been trivialised. The justification problem will reappear, however, in a disguised form, as the question 'Are there any deductively valid/inductively strong arguments?'. If, on the other hand, we adopt syntactic accounts of deductive validity/inductive strength, the nature of the justification problem is clear: to show that arguments which are deductively valid/inductively strong are also truth-preserving/truth-preserving most of the time (i.e. deductively valid/inductively strong on the semantic accounts). On the other hand, there is the difficulty that we must somehow specify which systems are possible values of ' $L_D$ ' and ' $L_I$ ', and this will presumably require appeal to inevitably vague considerations concerning the intentions of the authors of a formal system.

A convenient compromise is this. There are certain forms of inference, such as the rule:

$$RI \quad \text{From: } \frac{m}{n} \text{ of all observed } A\text{s have been } B\text{'s to infer: } \frac{m}{n} \text{ of all } A\text{'s are } B\text{'s}$$

which are commonly taken to be inductively strong, and similarly, certain forms of inference, such as

$$MPP \quad \text{From: } A \Rightarrow B, A \text{ to infer: } B$$
which are generally taken to be deductively valid. Analogues of the general justification problems can now be set up as follows:

the problem of the justification of induction: show that RI is truth-preserving most of the time.
the problem of the justification of deduction: show that MPP is truth-preserving.

My procedure will be, then, to show that difficulties arise in the attempt to justify MPP which are analogous to notorious difficulties arising in the attempt to justify RI.

(3) I consider first the suggestion that deduction needs no justification, that the call for a proof that MPP is truth-preserving is somehow misguided.

An argument for this position might go as follows:

It is analytic that a deductively valid argument is truth-preserving, for by 'valid' we mean 'argument whose premisses could not be true without its conclusion being true too'. So there can be no serious question whether a deductively valid argument is truth-preserving.

It seems clear enough that anyone who argued like this would be the victim of a confusion. Agreed, if we adopt a semantic definition of 'deductively valid' it follows immediately that deductively valid arguments are truth-preserving. But the problem was, to show that a particular form of argument, a form deductively valid in the syntactic sense, is truth-preserving; and this is a genuine problem, which has simply been evaded. Similar arguments show the claim, made e.g. by Strawson in [1952], p. 257, that induction needs no justification, to be confused.

(4) I argued in Section (1) that 'justifications' of deduction are liable either to be inductive and too weak, or to be deductive and circular. The former, inductive kind of justification has enjoyed little popularity (except with the Intuitionists? cf. Brouwer [1952]). But arguments of the second kind are not hard to find.

(a) Consider the following attempt to justify MPP:

A1 Suppose that 'A' is true, and that 'A ⊃ B' is true. By the truth-table for '⊃', if 'A' is true and 'A ⊃ B' is true, then 'B' is true too. So 'B' must be true too.

This argument has a serious drawback: it is of the very form which it is supposed to justify. For it goes:

A1' Suppose C (that 'A' is true and that 'A ⊃ B' is true). If C then D (if 'A' is true and 'A ⊃ B' is true, 'B' is true). So, D ('B' is true too.

The analogy with Black's 'self-supporting' argument for induction [1954] is striking. Black proposes to support induction by means of the argument:

A2 RI has usually been successful in observed instances.

RI is usually successful.
He defends himself against the charge of circularity by pointing out that this argument is not a simple case of question-begging: it does not contain its conclusion as a premiss. It might, similarly, be pointed out that A1' is not a simple case of question-begging: for it does not contain its conclusion as a premiss, either.

One is inclined to feel that A2 is objectionably circular, in spite of Black's defence; and this intuition can be supported by an argument, like Salmon's [1966], to show that if A2 supports RI, an exactly analogous argument would support a counter-inductive rule, say:

**RCI**  From: most observed A's have not been B's
          to infer: most A's are B's.
Thus:
A3  RCI has usually been unsuccessful in the past.
     . RCI is usually successful.

In a similar way, one can support the intuition that there is something wrong with A1', in spite of its not being straightforwardly question-begging, by showing that if A1' supports MPP, an exactly analogous argument would support a deductively invalid rule, say:

MM (modus morons);
From: A ⊃ B and B
     to infer: A.
Thus:
A4  Supposing that 'A ⊃ B' is true and 'B' is true, 'A ⊃ B' is true
     ⊃ 'B' is true.
Now, by the truth-table for '⊃', if 'A' is true, then, if 'A ⊃ B' is true, 'B' is true. Therefore, 'A' is true.

This argument, like A1, has the very form which it is supposed to justify. For it goes:

A4'  Suppose D (if 'A ⊃ B' is true, 'B' is true).
     If C, then D (if 'A' is true, then, if 'A ⊃ B' is true, 'B' is true).
     So, C ('A' is true).

It is no good to protest that A4' does not justify *modus morons* because it uses an *invalid* rule of inference, whereas A4' does justify *modus ponens*, because it uses a *valid* rule of inference—for to justify our conviction that MPP is valid and MM is not is precisely what is at issue.

Neither is it any use to protest that A1' is not circular because it is an argument in the meta-language, whereas the rule which it is supposed to justify is a rule in the object language. For the attempt to save the argument for RI by taking it as a proof, on level 2, of a rule of level 1, also falls prey to the difficulty that we could with equal justice give a counter-inductive argument, on level 2, for the counter-inductive rule at level 1. And similarly, if we may give an argument using MPP, at level 2, to support the rule MPP at level 1, we could, equally, give an argument, using MM, at level 2, to support the rule MM at level 1.

(b) Another way to try to justify MPP, which promises not to be
Thomson's argument is that Achilles should never have conceded that an extra premiss was needed; for, he argues, if the original inference was valid (semantically) the added premiss is true but not needed, and if the original inference was invalid (semantically) the added premiss is needed but false. There is an analogy, here, again, with attempts to justify induction by appending a premiss—something, usually, to the effect that 'Nature is uniform'—which turns inferences in accordance with RI into deductively valid inferences. The required premiss would, presumably, be true but not needed if RI were deductively valid, false but needed if it is not.

Thomson’s idea suggests that we should contrast this picture in the case of MPP:

A5  
(1) A ⊃ ((A ⊃ B) ⊃ B) (true but superfluous premiss)  
(2) A  
(3) (A ⊃ B) ⊃ B  
(4) A ⊃ B  
(5) B

with this picture in the case of MM:

A6  
(1) B ⊃ ((A ⊃ B) ⊃ A) (false but needed premiss)  
(2) B  
(3) (A ⊃ B) ⊃ A  
(4) A ⊃ B  
(5) A

Thomson's point is that in A5 premiss (1) is a tautology, so true; but it is not needed, since lines (2), (4) and (5) alone constitute a valid argument. In A6, by contrast, premiss (1) is not a tautology; but it is needed, because lines (2), (4) and (5) alone do not constitute a valid argument. But this is to assume that MPP, which is the rule of inference in virtue of which in A5 (2) and (4) yield (5), is valid; whereas MM, which is the rule of inference in virtue of which, in A6, (2) and (4) would yield (5), is not valid. But this is just what was to have been shown.

If A5 justifies MPP, which, after all, it uses, then the following argument equally justifies MM:

A7  
(1) (A ⊃ B) ⊃ (A ⊃ B) (true but superfluous premiss)  
(2) A ⊃ B  
(3) A ⊃ B  
(4) B  
(5) A

In A7 as in A5 the first premiss is a tautology, so true, but it is superfluous, since (if MM is accepted) lines (2), (4) and (5) alone constitute a valid argument.
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(c) Nor will it do to argue that MPP is, whereas MM is not, justified 'in virtue of the meaning of "\( \Rightarrow \)". For how is the meaning of "\( \Rightarrow \)" given? There are three kinds of answer commonly given: that the meaning of the connectives is given by the rules of inference/axioms of the system in which they occur; that the meaning is given by the interpretation, or, specifically, the truth-table, provided; that the meaning is given by the English readings of the connectives. Well, if "\( \Rightarrow \)" is supposed to be at least partially defined by the rules of inference governing sentences containing it (cf. Prior [1960], [1964]) then MPP and MM would be exactly on a par. In a system containing MPP the meaning of "\( \Rightarrow \)" is partially defined by the rule, from "\( A \Rightarrow B \)" and "\( A \)" to infer "\( B \)". In a system containing MM the meaning of "\( \Rightarrow \)" is partially defined by the rule, from "\( A \Rightarrow B \)" and "\( B \)", to infer "\( A \)". In either case the rule in question would be justified in virtue of the meaning of "\( \Rightarrow \)" finally, since the meaning of "\( \Rightarrow \)" would be given by the rule. If, on the other hand, we thought of "\( \Rightarrow \)" as partially defined by its truth-table (cf. Stevenson [1961]), we are in the difficulty discussed earlier (c above) that arguments from the truth-table to the justification of a rule of inference are liable to employ the rule in question. Nor would it do to appeal to the usual reading of "\( \Rightarrow \)" as "if... then...", not just because the propriety of that reading has been doubted, but also because the question, why "\( B \)" follows from "if A then B" and "\( A \)" but not "\( A \)" from "if A then B" and "\( B \)", is precisely analogous to the question at issue.

(d) Our arguments against attempted justification of MPP have appealed to the fact that analogous procedures would justify MM. So at this point it might be suggested that we can produce independent arguments against MM. (Compare attempts to diagnose incoherence in RCI.) In particular, it might be supposed that it is a relatively simple matter to show that MM cannot be truth-preserving, since with MM at our disposal we could argue as follows:

\[
\begin{align*}
A8 & \quad (1) \ (P \land \neg P) \Rightarrow (P \lor \neg P) \\
(2) & \quad P \lor \neg P \\
(3) & \quad P \land \neg P \quad 1, 2 \ MM
\end{align*}
\]

So that a system including MM would be inconsistent. (This idea is suggested by Belnap's paper on 'tonk'.)

However, this argument is inconclusive because it depends upon certain assumptions about what else we have in the system to which MM is appended—in particular, that (1) and (2) are theorems. Now certainly if a system contained (1) and (2) as theorems, then (3) could be derived by MM, and the system would be inconsistent; but a system allowing MM can hardly be assumed to be otherwise conventional. (After all, many systems lack "\( P \lor \neg P \)" as a theorem, and minimal logic also lacks "\( P \Rightarrow (\neg P \Rightarrow Q) \)."

(5) It might be suggested at this point that to direct our search for justification to a form of argument, or argument schema, such as MPP, is misguided, that the justification of the schema lies in the validity of its instances. So the answer to the question, 'What justifies the conclusion?'
is simply 'The premisses'; and the answer to the further question, what justifies the argument schema, is simply that its instances are valid.

This suggestion is unsatisfactory for several reasons. First, it shifts the justification problem from the argument schema to its instances, without providing any solution to the problem of the justification of the instances, beyond the bald assertion that they are justified. The claim that one can just see that the premisses justify the conclusion is implausible in the extreme in view of the fact that people can and do disagree about which arguments are valid. Second, there is an implicit generality in the claim that a particular argument is valid. For to say that an argument is valid is not just to say that its premisses and its conclusion are true—for that is neither necessary nor sufficient for (semantic) validity. Rather, it is to say that its premisses could not be true without its conclusion being true also, i.e. that there is no argument of that form with true premisses and false conclusion. But if the claim that a particular argument is valid is to be spelled out by appeal to other arguments of that form, it is hopeless to try to justify that form of argument by appeal to the validity of its instances. (Indeed, it is not a simple matter to specify of what schema a particular argument is an instance. Our decision about what the logical form of an argument may depend upon our view about whether the argument is valid.) Third, since a valid schema has infinitely many instances, if the validity of the schema were to be proven on the basis of the validity of its instances, the justification of the schema would have to be inductive, and would in consequence inevitably fail to establish a result of the desired strength. (Cf. Section 1.)

In rejecting this suggestion I do not, of course, deny the genetic point, that the codification of valid forms of inference, the construction of a formal system, may proceed in part via generalisation over cases—though in part, I think, the procedure may also go in the opposite direction. (This genetic point is, I think, related to the one Carnap [1968] is making when he observes that we could not convince a man who is 'deductively blind' of the validity of MPP.) But I do claim that the justification of a form of inference cannot derive from intuition of the validity of its instances.

(6) What I have said in this paper should, perhaps, be already familiar—it is foreshadowed in Carroll [1895], and more or less explicit in Quine [1936] and Carnap 1968 ('... the epistemological situation in inductive logic... is not worse than that in deductive logic, but quite analogous to it', p. 266). But the point does not seem to have been taken.

The moral of the paper might be put, pessimistically, as that deduction is no less in need of justification than induction; or, optimistically, as that induction is in no more need of justification than deduction. But however we put it, the presumption, that induction is shaky but deduction is firm, is impugned. And this presumption is quite crucial, e.g. to Popper's proposal [1959] to replace inductivism by deductivism. Those of us who are sceptical about the analytic/synthetic distinction will, no doubt, find these consequences less unpalatable than will those who accept it. And those of us who take a tolerant attitude to nonstandard logics—who regard logic as a theory, revisable, like other theories, in the light of experience—may even find these consequences welcome.
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